To Françoise,
her courage, quick wit and patience

*Why struggle to PROOVE it when
you can easily PLOUFFE it?*

Graeme Reeves

Foreword

It's a story about numbers, figures, tons of figures. I often say this to people who are surprised to see me give the total of the bill in a store, don't worry, I fell into numbers when I was a kid. Or scrolling through decimals in class when talking about π or $\sqrt{2}$. I sometimes write them on the blackboard, and I remember the first 200 a bit. A teacher at the IUT once said to me, you know Simon, your decimals of π it's contagious, now several of them learn them by heart. I once caught one of them with his eyes glued to the ceiling, in a

strange trance. He replied, "I'm learning decimals of π I'm up to 100.

Back in 1975, I knew 4096 of them, enough to set a world record that lasted 2 years and launched the decimal fad. Other mathematicians and even famous physicists got in on the act. Richard Feynman, the famous physicist, knew exactly 762. At this point in the decimal development of π 999999, he had fun saying yes, and after 762 it's all 9s, so much so that this particular point is called Feynman's point. He was right, since 24 decimal places of π are quite sufficient for all practical applications and even for NASA. But why calculate so many decimal places of this number? That's a good question. The answer depends on who's asking it. The real answer for me is far too complicated to explain to someone who doesn't see the universe we live in. The real answer is deeper, in fact, and began with Pythagoras 2,500 years ago. For Pythagoras, the very nature of 'everything' is number, and when they (the school of Pythagoras) came across the first irrational number, which is $\sqrt{2}$ it was a shock, a schism. How can a square with side 1 have a diagonal that isn't a whole or rational number? So we got interested in numbers a long time ago, and today the story is far from over, because we're beginning to understand. Pythagoras' schism with $\sqrt{2}$ is the same as that found for atoms and

particles at the very heart of matter. These mathematical constants are in fact very important. One in particular was identified by a certain Broadhurst in a corner of the electron. The 3ème magnetic moment of the electron is a dimensionless value. Dimensionless means that it is a pure number, which does not depend on terrestrial or human physical units. Its value is $Li_4(1/2)0.51747906167...$ and this same electron also has others of the same nature. True pure mathematical constants at the very heart of the fabric of this universe we live in? I've been convinced of this for many years.

So, this answer to give to someone who asks, what's the point of your calculations? Now I'd rather say, in fact you've got how much time ahead of you, I'll explain. Welcome to the Sea of Real Numbers.

Note: the term "mathematician" is used in this book in the masculine form for the sake of simplicity. Women in mathematics are certainly as good as men, and often far humbler.

Note 2: the book contains as few formulas as possible. The non-mathematician reader can skip these formulas, which only appear to support an argument. For those who want to know more, all my articles can be found here: http ://plouffe.fr/articles/ and http://plouffe.fr/simon/Citations/

Note 3: there's an old saying in the publishing world: "For every formula you add to a text, 50% of readers drop out". So if I write a book with 50 formulas in it, there will probably be 1 reader left interested in the book at the end. So good luck with that.

Chapter 1

The red mill

St-Jovite, a village of 2,000 inhabitants in Quebec's Laurentians, is where my parents settled in 1949 in an old mill.

The mill circa 1950

They were weavers, and my mother in particular traded in curtains, bedspreads and anything else that could be woven. It was called *cottage industry*, and the employees were mostly women who wove on their Leclerc looms (very popular at the time), made of very hard wood. This

little business was doing well, as the St-Jovite region was hosting an automobile race and the Mont-Tremblant ski resort was very popular. It brought in a lot of American tourists.

My parents in 1950 or thereabouts. It's in front of the house.

So, along the way, my parents had many children: 8 in all, and I'm the 5th . Inevitably, the house had to be extended, which they quickly did.

The enlarged mill circa 1954

Aerial view (1990)

The mill in 1954

Around 1965, they decided not only to enlarge the house but also to install an indoor swimming pool. My mother said it would keep us busy, and so it did. We spent a lot of time there, winter and summer alike, since the room was also heated by a huge fireplace, and we'd stand in front of it to dry off after a dip in the water.

Our favorite game, apart from the pool, was hide-and-seek, but we often couldn't find each other - the number of nooks and crannies in this house was enough to discourage the search - over 600 square meters. When we weren't taking a dip in the pool, we went skiing, which we could do by putting our skis on the gallery, skiing down the stairs and heading for the mountains. In summer, it was the stream next to the house or the mountains, the forest all around. This place was a paradise.

We'd make tomato sandwiches and picnic on the mountain, topped by a cross like the one on Montreal's Mont Royal, from which we could contemplate the village and, of course, the house. Some of these photos were found on the official website of the Bibliothèque nationale du Québec, while searching the archives.

Everything was original in this house, and the business my parents had created occupied around twenty families around the village. They were all equipped with looms. Madame and Monsieur Perrault from Brébeuf made the woven wool rugs, Madame Gauthier did the sewing and weaving, and Titi or Mercédes from Saint-Jovite Station (2-3 km from the village) sewed the woven curtains. Everything was woven or braided in the house, and in the enormous cellar there was the carpet room, the thread spool room and the warping room, where my mother spent hours braiding skeins of thread for the looms. It was here that I nearly lost a thumb when I got my fingers caught in the motor pulley. I don't even remember that episode.

The house had 4 levels: the cellar, which covered the entire basement, contained the carpet room, the garden shed, the yarn room, where cones and spools of cotton and wool yarn were stored for

making hangings and curtains. Finally, the garage was used mainly for dyeing wool for carpets.

Upstairs, the store and the rest of the house up to the star chamber. The rooms had been given names. The star chamber was the little room where Maria, the eldest, had her bedroom next to the wheel, a real mill wheel that turned with the wind. We turned it by hand for fun.

The main floor was cut off from the store by a large curtain that opened onto the kitchen. It was at about this point that the railroad track was laid, which served as a support beam for all this wood; the whole house was made of wood. The mix of business and home was strange at times, once a lost (?) customer showed up in the kitchen and threw a dime on the counter and asked for a coffee. It was explained to him that this wasn't a restaurant and that he was in the kitchen of the house.

In front of the swings, before the indoor pool was built, there was the outdoor pool in the shape of a soup bowl. In order, Domino, Jacinthe, Lucie, Nicolas, Loulou, Sonia and me.
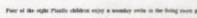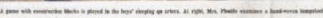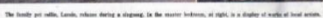
The house in winter, you can see my parents' bedroom. This rather special shape, the only very high room with this stained-glass window. At the back we see the chambre aux étoiles, Maria's room, the eldest in the family, where she had set up her headquarters.

The whole family and 2 friends.

Montreal Star article, August 27, 1966

My parents and our dog Lassie
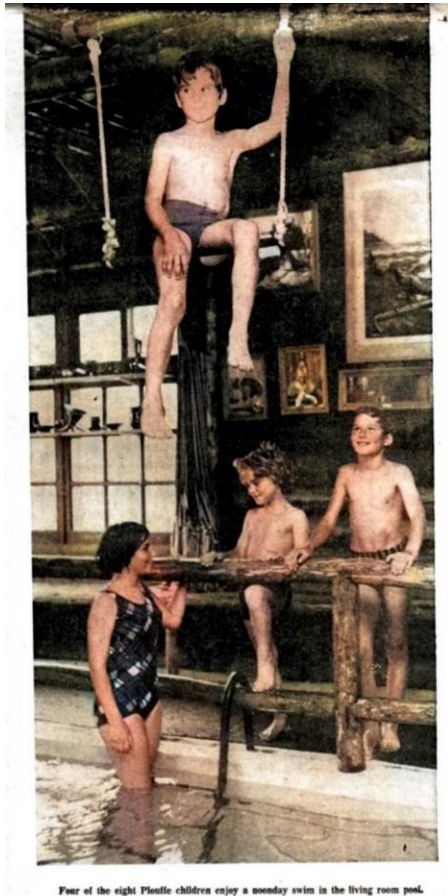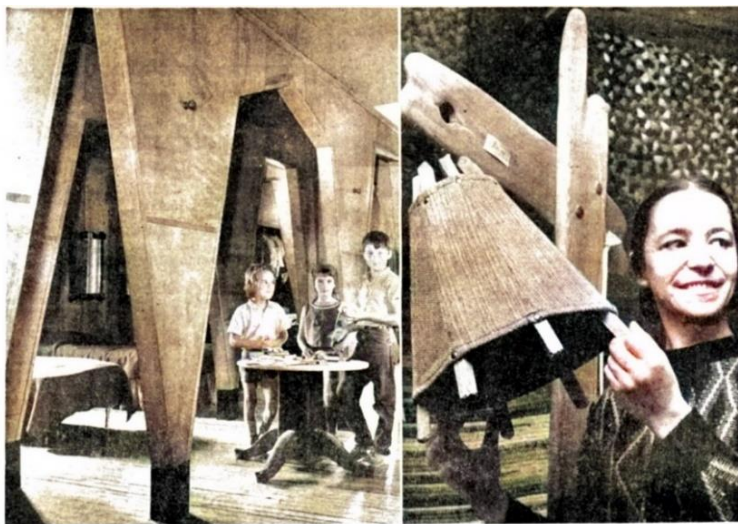
Four of the eight Plouffe children enjoy a noonday swim in the living room pool.

Me on the trapeze with Lucie, Nicolas and Domino.

A game with construction blocks is played in the boys' sleeping qu arters. At right, Mrs. Plouffe examines a hand-woven lampshade.

Excerpt from the Montreal Star, August 27, 1966

On the loom, in the 70s at the boutique on rue St-Paul in old Montreal

There was never a dull moment at home. We spent our days in that pool, hanging on to trapezes and ladders around the pool, swinging on the swings inside. It was a pretty big room. My parents used to organize song clubs there on Saturdays, and around the pool there were benches where spectators could sit. It attracted a lot of customers, and my parents' business was well-known in those days. A Montreal newspaper, the *Montreal Star,* did an article on the house, the business and my parents. It was catchy because our name was Plouffe, and the Plouffe family (Roger Lemelin's TV program and novel) had been very popular in Quebec and even elsewhere in the world.

National television (Radio-Canada) had come to the house to interview my mother. We had a full insert in La Presse that same year. TV crews once came to the house with 4 big trucks to film a soap opera. The house was used as a set for the show. They invaded the house with cameras, wires, dollies, big swivel chairs for make-up and technical crews for a whole weekend, ending up with not even 2 minutes of film. There was even someone in a diving suit in the pool to film the stuntman (a woman) diving into the water, as they wanted to film her in the water. The water in the pool was too cold for the actress and, I imagine, there was also the danger of an accident. We had a good laugh about it. The scene lasted 5 seconds and took all day to film. I remember that the main actor had 1 sentence to say and he repeated it I don't know how many times. This soap opera (Rue des Pignons) was popular in Quebec at the

time, I even think it was the most popular. What's more, my uncle Pierre Brabant, who was a concert pianist, composed the music for the show. There must have been 5 of us out of the 8 children who played, including me. My mother had bought a Hammond organ, I think, and played it sometimes, as well as taking correspondence courses in mathematics. My older sister Maria played guitar instead. Music was very important at home, and we listened to a bit of everything. All that music left a very strong impression on me. Brassens, Bécaud, Michel Legrand, the Swingle Singers. My father only listened to classical music and hated my sister Maria's 'yé-yé' records, but that was all right. For me, this very happy childhood was, now that I'm older, an idea of heaven on earth. At school, I don't remember getting less than 95 out of 100 on my report cards, but arithmetic was my favorite. I skipped grade 5 and went from grade 4 to grade 6 . In Quebec, years are counted like floors. Primary school is from 1 to 7, secondary school is from I to V, then Cégep, which lasts 2 years and would be the equivalent of preparatory classes in France, and then university. My mother often told me this: when you were 2 years old, you didn't say anything at all, you were autistic, my grandfather used to say, he's thinking. I had quite severe meningitis when I was 5 and I'm more or less normal now, but I could have stayed there. My sister Jacinthe used to call me the priest because I often walked around the pool thinking, and once when I was about 10, after much reflection, I said to my mother: *Everything that is logical is possible,* although I had no idea how I could have come to that conclusion.

I had a tendency towards numbers, and I remember at the age of 8 handwriting numbers from 1 to 1001 just to see. In elementary school, I only liked the afternoons when we did arithmetic, addition, multiplication and division. I knew the multiplication table by heart, of course.

But actually, it's a bit more than that. When I was about 10, I received LEGOs as a present. But before I had LEGOs, my parents gave me cardboard rolls and cones to keep me busy, and I made houses out of them. Other cardboard boxes came from my grandfather, left over from the cutting of his printing press in Montreal.

With these LEGOs, I'd spend hours building a tower taller than me. I wanted to be an architect in life. The big cellar at home was filled with spools of thread of all colors, and my parents obviously had a business in curtains, carpets and so on. That was the stock. I'd invented a game of cable cars with LEGOs, putting wires everywhere, in the house, outside. I covered the parking lot with wires stretched out in all directions. What for? I don't even know, but shooting straight lines into space with these wires fascinated me, even though using Lego cable cars didn't work over long distances.

I used to clutter up the parking lot with my constructions, so much so that store customers had to do acrobatics to get around my wires, not daring to demolish my work. I had little access to books: we had the National Geographic that my godfather brought and that my father read in English, and the Larousse with the

few pictures, and that was all. It wasn't until 1968 that I received the TIME-LIFE science books on mathematics and physics. These books were well done, and I had read them all. My hero was of course Einstein. For me, the idea of infinity became obvious when I saw the sea for the first time in 1964. We'd been to Virginia, and when I got to the edge of the sea, I could see it just to see it, until the end of a wave came to my feet. It was a shock to see such immensity.

I had a few memories of my father after the family moved to Montreal. When we were very young, he was always tinkering around the house with a hammer, and I was always asking him questions, so many questions that he'd end up saying, "You're annoying me! He was the one who did the woodwork, while my mother was rather busy with the 'les artisanes enrg' business. She spent her time making curtains - I say curtains, but we called them drapes. We had several looms in the house, and all the craftswomen, mostly women, had one or more, and they wove. I remember that my mother made it her honor and duty to pay them well. It wasn't a fortune, but they were very happy to make a good income weaving at home. There must have been about twenty people working in the village and surrounding area. At home, my parents also wove rugs when they had the time. These narrow, very long rugs were sold by the yard (36 inches). With the many American tourists, my parents spoke English all the time, which is how I learned English. These American tourists were a bit exotic for us, and I'd have fun

crawling around on the floor and tying their shoelaces together.

My parents divorced in 1968 and the family, almost the whole family, moved to Montreal (Nun's Island). The mill was finally sold several years later by my father, who I believe sold it for a pittance. The building was repainted, and since it was already red, they added another coat. It became a discotheque, Le Moulin Rouge, but nothing like the one in Paris. I went there once to see the house and what it had become. It's weird to see a cash machine installed in the kitchen. The swimming pool covered and transformed into a dance floor.

In Montreal, I went to high school by bus, which was a shock. My grades, which were close to 100%, dropped to 50%, I didn't understand anything, I had been given a computerized timetable on a sheet of paper and I didn't even understand the timetable. I remember my accounting teacher saying to me, 'Are you plugged up, Plouffe?' I didn't see the point of assets and liabilities, not at all.

It was there that I began to study physics myself, reading the encyclopedias Universalis and Britannica. I spent most of my time at the Ville Mont-Royal library, not too far from home. You had to take the bus to get there, but taking the bus every time was a pilgrimage for me. Once I got in, I'd stay all day on Saturdays. I was so into it that I ended up not going to school at all - I didn't

go to classes anyway, I'd decided that classes were useless to me, that I'd learn much more from the Grand Larousse Encyclopédique or Universalis, I'd read what I wanted and above all I'd started doing calculations. It was at this point that I really began to love classical music and discovered Bach and Telemann. I'd read that Telemann was an autodidact and of course Bach, with the Brandenburg concertos that I listened to over and over again. But Telemann was my favorite, and the music put me in a second state, an effect I still have today every time I listen to it.

I ended up dropping out of school completely at the age of 16-17. The principal had given up on the idea of forcing me to attend classes, especially as, with my calculator and a logarithm table, I could calculate anything. I also had a few classic slide rules, but to 2 decimal places they were of little interest. I still have this logarithm table, all daubed up, it's my first math book, in fact. I used to wonder, but how would you calculate in columns the numbers that appeared in rows?

Chapter 2

The calculator

June 11, 1971, I'm 15 years old. I receive a camera as a present, the Praktika Nova IB. But what's the point of a thing like that? Oh yes, to take photos, but you have to develop them, which bored me terribly. My godfather Maurice Dury had just bought himself an electronic calculator, a Bowmar 4 operations with 8 decimal places. We agreed to make the swap, even though the camera was worth much more. It was love at first sight.

Bowmar 905, 4 operations

I remember buying several mechanical and electromechanical calculating machines shortly afterwards, wanting to understand how they calculated. I'd managed to take them apart, but understanding the mechanism completely eluded me.

Armed with this calculator, I set about calculating the inverses of prime numbers. I wanted to break the barrier of the 8-digit display. I had found a way to calculate the other digits that followed. Knowing how to do long division by hand, I simply adapted what I knew for the calculator. I had made myself a table, which must have been in 1972-73. So, 1/7 = 0.142857... had 6 digits in its period. 1/19 had 18 digits, 1/17 had 16 digits but 1/13 had only 6 digits. Looking in the college library (Collège Saint-Viateur), I found the famous 'Que sais-je', a collection of little books on all sorts of subjects, and one on number theory that explained this phenomenon. For me, explaining meant that somewhere along the line I'd understood that in base 10, the inverse of primes is either $p - 1$ or a submultiple of $p - 1$. For example, 1/37 has 3 digits (0,027027027...) in its period and 3 divides 36 which is $p - 1$. But what puzzled me was why some primes have a period of $p - 1$ while others have less. I realized that this phenomenon is independent of the base; if we calculate in base 2, the periods are different but just as mysterious. As far as I know, this is an unsolved problem in mathematics.

In 1974, I was trying to understand why the decimals of 1/17 are predictable and why those of π are not. Predictable meaning that you could calculate in advance. In fact, 1/17 = 0.0588235294117647... and it repeats indefinitely. But with each decimal obtained, we obtain the residue of the long division of 1/17 - yes, because it's 1.000000000... which we divide by 17. Or, more mathematically, the residue of 10^n modulo 17 is