

Les chiffres

1, 2, 3, 5 et la suite

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Assez difficile de faire plus numérique que des chiffres...

- ▶ Au début, au siècle des lumières (1700-1800), la science mathématique subissait une métamorphose avec la découverte du calcul numérique.
- ▶ On s'est aperçu qu'en tabulant une fonction on pouvait prédire quelque chose.
- ▶ Mais avant on faisait comment ? Essentiellement des rapports géométriques issus de la géométrie classique (Euclide et autres grecs).
- ▶ C'est au courant de 1600 que sont apparues les premières tables de logarithmes, et à peu près à la même époque les règles à calcul.

Table de logarithme ? Règle à Calcul ?

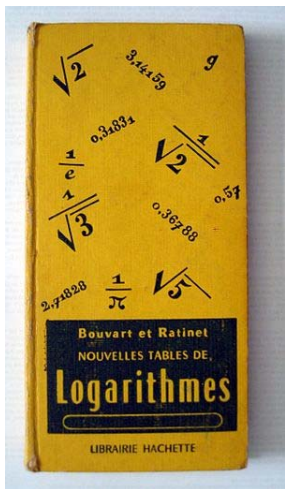


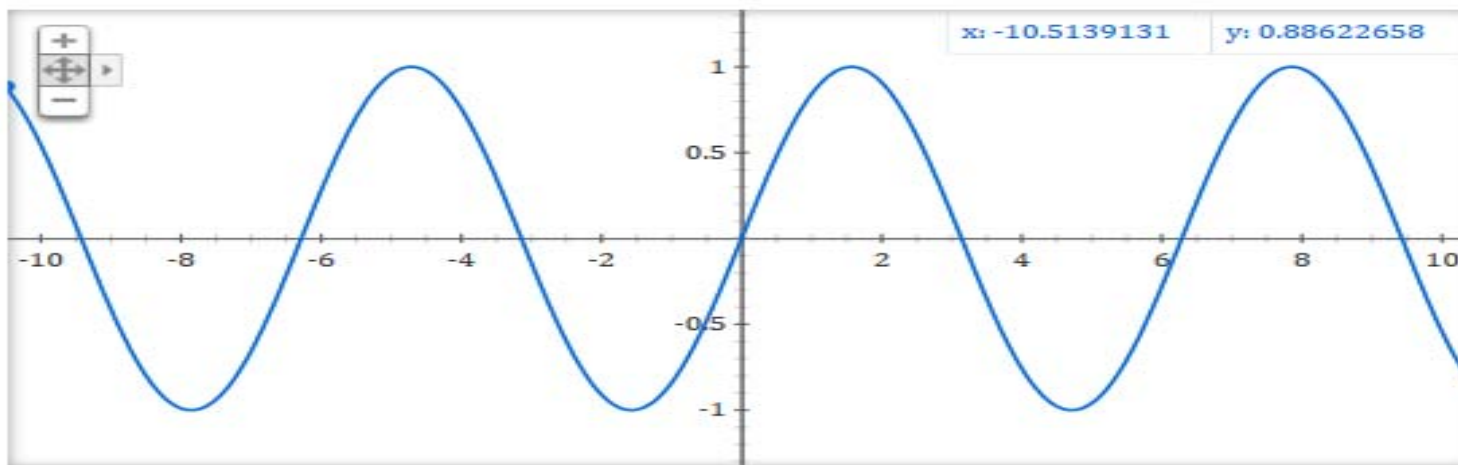
Table de logarithme Bouvart et Ratinet (1905) et encore utilisée en 1980
Napier 1614 d'où le nom de logarithmes népériens.

N	0	1	2	3	... 9
10	0000	0043	0086	0128	...
11	0414	0453	0492	0531	...
12	0792	0828	0864	0899	...
13	1139	1173	1206	1239	...
14	1461	1492	1523	1553	...
15	1761	1790	1818	1847	...
16	2041	2068	2095	2122	...
17	2304	2330	2355	2380	...
18	2553	2577	2601	2625	...
19	2788	2810	2833	2856	...



On les utilisait pour les calculs astronomiques, financiers.

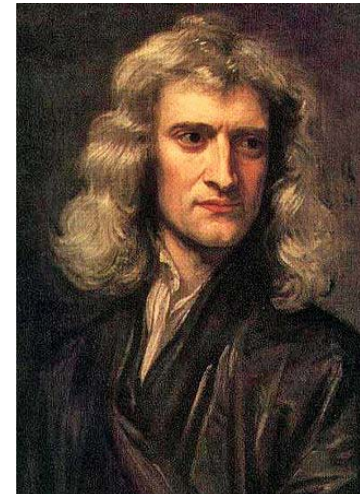
- ▶ Donc on savait calculer une fonction comme sinus



Mais c'est Newton qui a systématiquement mathématisé le procédé

► Différences de Newton

i	x_i	f_i	Δf_i	$\Delta^2 f_i$	$\Delta^3 f_i$	$\Delta^4 f_i$
0	1.0	.0				
1	3.0	1.0986	1.0986	-0.5878	0.4135	
2	5.0	1.6094	0.5108	-0.1743		-0.3244
3	7.0	1.9459	0.3365	-0.0852	0.0891	
4	9.0	2.1972	0.2513			



Et puis arriva l'analyse mathématique

Handwritten mathematical notes covering various topics in analysis, including calculus, geometry, and algebra. The notes are densely packed with formulas and diagrams.

Calculus and Algebra:

- Trigonometric identities: $\cos x = \frac{1-t^2}{1+t^2}$, $\sin 2x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$
- Integration techniques: $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x$, $\int \frac{1}{1+x^2} dx = \arctan x$, $\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin \frac{x}{a}$
- Differentiation: $\frac{d}{dx} \arcsin \frac{x}{a} = \frac{1}{\sqrt{a^2-x^2}}$, $\frac{d}{dx} \arctan \frac{x}{a} = \frac{1}{a^2+x^2}$
- Series expansion: $f(x) \sim \frac{a_0}{2} - \sum_{n=1}^{\infty} a_n \cos nx - b_n \sin nx$

Geometry:

- Diagrams of a sphere, a cylinder, and a cone.
- Formulas for surface area and volume: $S_1 = 4\pi R^2$, $V = \frac{4}{3}\pi R^3$, $S = \pi R^2$
- Coordinate systems and vector algebra: $\vec{a}_1, \vec{a}_2, \vec{a}_3 = \vec{0}$, $\vec{a}_1 + \vec{a}_2 + \vec{a}_3 = \vec{0}$

Other Mathematical Concepts:

- Complex numbers: $z = e^{i\theta} = \cos \theta + i \sin \theta$
- Logarithms: $\log \frac{1}{2} x$, $\log \frac{1}{2} x$
- Trigonometric functions: $\sin 2x$, $\cos 2x$, $\sin 3x$, $\cos 3x$

$0 - 2 - x^2 - 12x + 13 = 0$ $\alpha = \frac{R^2}{c^2}$ e^x $c' = 0$; $13C - x^2 + 13 - 2x^2 - 12x^3$ $x' = 1$; $2(x^3)' = 2 \cdot 3x^2$; $y = 2x^3 - 5x^2 + 7x + 4 - 2$
 $d = 13C - x \ln a^2$ (e^x) $(x^2 + 13)^2 - x^4$ $[-5; 5]$ $1 - \frac{u^2}{8u^2} \ln((x-a)^{n+1})^2$; $\frac{2-t}{2+t^2}$ $\alpha = \frac{R^2}{c^2}$ $V = \frac{1}{3}\pi R^2 h$
 $y = \frac{(x-16)}{(x^2+138)}$ $r_n(x) = \ln a^2$; $\frac{R^2}{c^2} \ell$ $c' = 0$; $r_n(x) = \ln a^2(x^5)$ $C = \frac{a^2 b^2 c^2 - d^2}{(d+a)^2} (3R-h)$
 $\int 2x' = 2xy + 1 = 0$, $e^3 = x \ln a^2 \cdot d \cdot 3x^2 + 13C$; \vec{n} $2xe^x = (e^x)' \cdot e^x$; $\frac{t^2}{2+t^2}$; $2xe^x = (e^x)' \cdot e^x$;
 $\int 2xy = 2y - x + 1 = 0$; $\int \sin(2-3x) dx$ $x=1$ $y = \frac{(x-16)}{(x^2+138)}$ $y = \frac{t^2}{2+t^2}$; $2xe^x = (e^x)' \cdot e^x$;
 $S = \lim \sum T_i = \lim \sum \frac{R_i}{|\cos \theta_i|} \frac{e^{2x} - 1}{(e^x - 1)^{1/4}} \frac{1}{8u^2}$ $f = \frac{2\ell^3}{EIu^4} (d+a)^2$ $y' = (2x)^3 - (5x)^3 - (7x^3)' + (4)' + 2x^3$;
 $w = (s, \rho, s) = \frac{m}{r} \iint \frac{dx dy}{|\cos \theta|}$ $\frac{e}{4\sqrt{e^x+1}}$ A_2 $f = \frac{2\ell^3}{EIu^4} (d+a)^2$ $y' = (2x)^3 - (5x)^3 - (7x^3)' + (4)' + 2x^3$;
 $w = (s, \rho, s) = \iint \frac{\rho ds}{r}$ $\frac{4}{16}$ A_1 A_2 A_3 A_4 $3x \cdot (8x) \cdot \sin x^2$ $\sqrt{A_2}$ $8y^{2x} + x^3 - 16$ $2xe^x = (e^x)' \cdot e^x$;
 $\frac{-2(-2)}{-2^2}$ $3x$ $\int \frac{dx}{6-3x}$ $\frac{3 \cos 3x}{3}$; $\int \frac{\sin 3x}{\cos^2 3x} 2x$; $15m^3$; S_1, m_1 3 $y' = \frac{(1)'(x^2) - (x^2) \cdot 1}{(x^2)^2}$;
 $\int \frac{e^{2x}}{(e^x+1)^{1/2}} dx$ α_2 $\sqrt{\tan 4x}$ $\int \cos(2+3x) dx$ $\frac{1}{2 \cos^2 x}$ $\vec{j} = y$ $6-1$;
 $\int \frac{dx}{3-x}$; $\int_0^2 \frac{x^2}{\sqrt{16-x^2}} dx$; $\left(\frac{1}{n+2}\right)^3 \frac{1}{12} S^2 \frac{3x^3 \ell \sqrt{n}}{\ln a}$ $S = \sum_{i=1}^n S_i, m_i$ $x = \frac{2-t}{2+t^2}$ $\sum \left(\frac{1!}{n-2}\right)^3 + \sum \left(\frac{1!}{2+18x^2}\right)^3$;
 $\int \sin(3-2x) dx$; $\int \frac{\cot^4 3x}{\sin^2 3x} dx$; $\sin^2 3x$ $S = \sum_{i=1}^n S_i, m_i$ $\int \frac{dx}{\cos^2 4x \sqrt{\tan 4x}}$ $(3R-h)$;
 $\frac{1}{2 \cos^2 x}$ S^2 $\int \frac{\cot^4 3x}{\sin^2 3x} dx$; $M = S + 14t^2 + 15m^3$; $\int \int f(x) dx$ $\int \frac{\cot^4 3x}{\sin^2 3x} dx$; $\frac{-2x}{x^4}$;
 $\frac{1}{12}$ $S = m_2$ $x = 4 + 3t$ $\int \sin(2-3x) dx$ $\int \frac{\cot^4 3x}{\sin^2 3x} dx$; $\frac{3x^2}{e^x}$;
 $y = e^x$ $3x^2$ $\frac{-2}{1} = -2 < 0$; $y_1 = \frac{1}{x^2}$ $E = \frac{\rho}{s n \epsilon^5}$; $S = \sum_{i=1}^n S_i, m_i$ $\frac{x}{12} t^2 \ln x^{\frac{1}{2}}$ $\frac{-2x}{x^4}$ $y' = \frac{(1)'(x^2) - (x^2)' \cdot 1}{(x^2)^2}$;
 $2x^3 - 5x^2 - 7x + 4 - 2$ $\frac{-2}{1} = -2 < 0$; $y_1 = \frac{1}{x^2}$ $E = \frac{\rho}{s n \epsilon^5}$; $S = \sum_{i=1}^n S_i, m_i$ $\frac{x}{12} t^2 \ln x^{\frac{1}{2}}$ $(-\infty, 0)$; $V = \frac{1}{3}\pi R^2 h$ $\alpha = \frac{\pi-2}{\pi}$;
 $+7x + 4 - 2$; $\lim_{x \rightarrow \infty} \frac{x^3}{e^x} = \frac{\infty}{\infty} \Rightarrow \frac{3x^2}{e^x}$ $(\tan 2x)' = \frac{1}{\cos^2 x} = \frac{1}{2 \cos^2 x}$ $(\sin 3x)' = (\sin)' 3x = (3x)' \sin x$

$\Delta y = (x + \Delta x)^2 - x^2 = 2x\Delta x + \Delta x^2$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (2x + \Delta x) = 2x$
 $(x^2)' = 2x$

$\Delta y = (x + \Delta x)^3 - x^3 = 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (3x^2 + 3x\Delta x + \Delta x^2) = 3x^2$
 $(x^3)' = 3x^2$

$\Delta y = (x + \Delta x)^4 - x^4 = 4x^3\Delta x + 6x^2\Delta x^2 + 4x\Delta x^3 + \Delta x^4$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (4x^3 + 6x^2\Delta x + 4x\Delta x^2 + \Delta x^3) = 4x^3$
 $(x^4)' = 4x^3$

$\Delta y = (x + \Delta x)^5 - x^5 = 5x^4\Delta x + 10x^3\Delta x^2 + 10x^2\Delta x^3 + 5x\Delta x^4 + \Delta x^5$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (5x^4 + 10x^3\Delta x + 10x^2\Delta x^2 + 5x\Delta x^3 + \Delta x^4) = 5x^4$
 $(x^5)' = 5x^4$

$\Delta y = (x + \Delta x)^6 - x^6 = 6x^5\Delta x + 15x^4\Delta x^2 + 20x^3\Delta x^3 + 15x^2\Delta x^4 + 6x\Delta x^5 + \Delta x^6$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (6x^5 + 15x^4\Delta x + 20x^3\Delta x^2 + 15x^2\Delta x^3 + 6x\Delta x^4 + \Delta x^5) = 6x^5$
 $(x^6)' = 6x^5$

$\Delta y = (x + \Delta x)^7 - x^7 = 7x^6\Delta x + 21x^5\Delta x^2 + 35x^4\Delta x^3 + 35x^3\Delta x^4 + 21x^2\Delta x^5 + 7x\Delta x^6 + \Delta x^7$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (7x^6 + 21x^5\Delta x + 35x^4\Delta x^2 + 35x^3\Delta x^3 + 21x^2\Delta x^4 + 7x\Delta x^5 + \Delta x^6) = 7x^6$
 $(x^7)' = 7x^6$

$\Delta y = (x + \Delta x)^8 - x^8 = 8x^7\Delta x + 28x^6\Delta x^2 + 56x^5\Delta x^3 + 56x^4\Delta x^4 + 28x^3\Delta x^5 + 8x^2\Delta x^6 + \Delta x^8$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (8x^7 + 28x^6\Delta x + 56x^5\Delta x^2 + 56x^4\Delta x^3 + 28x^3\Delta x^4 + 8x^2\Delta x^5 + \Delta x^7) = 8x^7$
 $(x^8)' = 8x^7$

$\Delta y = (x + \Delta x)^9 - x^9 = 9x^8\Delta x + 36x^7\Delta x^2 + 84x^6\Delta x^3 + 84x^5\Delta x^4 + 36x^4\Delta x^5 + 9x^3\Delta x^6 + \Delta x^9$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (9x^8 + 36x^7\Delta x + 84x^6\Delta x^2 + 84x^5\Delta x^3 + 36x^4\Delta x^4 + 9x^3\Delta x^5 + \Delta x^8) = 9x^8$
 $(x^9)' = 9x^8$

$\Delta y = (x + \Delta x)^{10} - x^{10} = 10x^9\Delta x + 45x^8\Delta x^2 + 126x^7\Delta x^3 + 126x^6\Delta x^4 + 54x^5\Delta x^5 + 10x^4\Delta x^6 + \Delta x^{10}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (10x^9 + 45x^8\Delta x + 126x^7\Delta x^2 + 126x^6\Delta x^3 + 54x^5\Delta x^4 + 10x^4\Delta x^5 + \Delta x^9) = 10x^9$
 $(x^{10})' = 10x^9$

$\Delta y = (x + \Delta x)^{11} - x^{11} = 11x^{10}\Delta x + 55x^9\Delta x^2 + 165x^8\Delta x^3 + 165x^7\Delta x^4 + 77x^6\Delta x^5 + 11x^5\Delta x^6 + \Delta x^{11}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (11x^{10} + 55x^9\Delta x + 165x^8\Delta x^2 + 165x^7\Delta x^3 + 77x^6\Delta x^4 + 11x^5\Delta x^5 + \Delta x^{10}) = 11x^{10}$
 $(x^{11})' = 11x^{10}$

$\Delta y = (x + \Delta x)^{12} - x^{12} = 12x^{11}\Delta x + 66x^{10}\Delta x^2 + 220x^9\Delta x^3 + 220x^8\Delta x^4 + 99x^7\Delta x^5 + 12x^6\Delta x^6 + \Delta x^{12}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (12x^{11} + 66x^{10}\Delta x + 220x^9\Delta x^2 + 220x^8\Delta x^3 + 99x^7\Delta x^4 + 12x^6\Delta x^5 + \Delta x^{11}) = 12x^{11}$
 $(x^{12})' = 12x^{11}$

$\Delta y = (x + \Delta x)^{13} - x^{13} = 13x^{12}\Delta x + 78x^{11}\Delta x^2 + 273x^{10}\Delta x^3 + 273x^9\Delta x^4 + 143x^8\Delta x^5 + 13x^7\Delta x^6 + \Delta x^{13}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (13x^{12} + 78x^{11}\Delta x + 273x^{10}\Delta x^2 + 273x^9\Delta x^3 + 143x^8\Delta x^4 + 13x^7\Delta x^5 + \Delta x^{12}) = 13x^{12}$
 $(x^{13})' = 13x^{12}$

$\Delta y = (x + \Delta x)^{14} - x^{14} = 14x^{13}\Delta x + 91x^{12}\Delta x^2 + 378x^{11}\Delta x^3 + 378x^{10}\Delta x^4 + 196x^9\Delta x^5 + 14x^8\Delta x^6 + \Delta x^{14}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (14x^{13} + 91x^{12}\Delta x + 378x^{11}\Delta x^2 + 378x^{10}\Delta x^3 + 196x^9\Delta x^4 + 14x^8\Delta x^5 + \Delta x^{13}) = 14x^{13}$
 $(x^{14})' = 14x^{13}$

$\Delta y = (x + \Delta x)^{15} - x^{15} = 15x^{14}\Delta x + 105x^{13}\Delta x^2 + 420x^{12}\Delta x^3 + 420x^{11}\Delta x^4 + 210x^{10}\Delta x^5 + 15x^9\Delta x^6 + \Delta x^{15}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (15x^{14} + 105x^{13}\Delta x + 420x^{12}\Delta x^2 + 420x^{11}\Delta x^3 + 210x^{10}\Delta x^4 + 15x^9\Delta x^5 + \Delta x^{14}) = 15x^{14}$
 $(x^{15})' = 15x^{14}$

$\Delta y = (x + \Delta x)^{16} - x^{16} = 16x^{15}\Delta x + 120x^{14}\Delta x^2 + 560x^{13}\Delta x^3 + 560x^{12}\Delta x^4 + 280x^{11}\Delta x^5 + 16x^{10}\Delta x^6 + \Delta x^{16}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (16x^{15} + 120x^{14}\Delta x + 560x^{13}\Delta x^2 + 560x^{12}\Delta x^3 + 280x^{11}\Delta x^4 + 16x^{10}\Delta x^5 + \Delta x^{15}) = 16x^{15}$
 $(x^{16})' = 16x^{15}$

$\Delta y = (x + \Delta x)^{17} - x^{17} = 17x^{16}\Delta x + 136x^{15}\Delta x^2 + 680x^{14}\Delta x^3 + 680x^{13}\Delta x^4 + 340x^{12}\Delta x^5 + 17x^{11}\Delta x^6 + \Delta x^{17}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (17x^{16} + 136x^{15}\Delta x + 680x^{14}\Delta x^2 + 680x^{13}\Delta x^3 + 340x^{12}\Delta x^4 + 17x^{11}\Delta x^5 + \Delta x^{16}) = 17x^{16}$
 $(x^{17})' = 17x^{16}$

$\Delta y = (x + \Delta x)^{18} - x^{18} = 18x^{17}\Delta x + 153x^{16}\Delta x^2 + 765x^{15}\Delta x^3 + 765x^{14}\Delta x^4 + 378x^{13}\Delta x^5 + 18x^{12}\Delta x^6 + \Delta x^{18}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (18x^{17} + 153x^{16}\Delta x + 765x^{15}\Delta x^2 + 765x^{14}\Delta x^3 + 378x^{13}\Delta x^4 + 18x^{12}\Delta x^5 + \Delta x^{17}) = 18x^{17}$
 $(x^{18})' = 18x^{17}$

$\Delta y = (x + \Delta x)^{19} - x^{19} = 19x^{18}\Delta x + 171x^{17}\Delta x^2 + 855x^{16}\Delta x^3 + 855x^{15}\Delta x^4 + 414x^{14}\Delta x^5 + 19x^{13}\Delta x^6 + \Delta x^{19}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (19x^{18} + 171x^{17}\Delta x + 855x^{16}\Delta x^2 + 855x^{15}\Delta x^3 + 414x^{14}\Delta x^4 + 19x^{13}\Delta x^5 + \Delta x^{18}) = 19x^{18}$
 $(x^{19})' = 19x^{18}$

$\Delta y = (x + \Delta x)^{20} - x^{20} = 20x^{19}\Delta x + 190x^{18}\Delta x^2 + 960x^{17}\Delta x^3 + 960x^{16}\Delta x^4 + 475x^{15}\Delta x^5 + 20x^{14}\Delta x^6 + \Delta x^{20}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (20x^{19} + 190x^{18}\Delta x + 960x^{17}\Delta x^2 + 960x^{16}\Delta x^3 + 475x^{15}\Delta x^4 + 20x^{14}\Delta x^5 + \Delta x^{19}) = 20x^{19}$
 $(x^{20})' = 20x^{19}$

$\Delta y = (x + \Delta x)^{21} - x^{21} = 21x^{20}\Delta x + 210x^{19}\Delta x^2 + 1029x^{18}\Delta x^3 + 1029x^{17}\Delta x^4 + 500x^{16}\Delta x^5 + 21x^{15}\Delta x^6 + \Delta x^{21}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (21x^{20} + 210x^{19}\Delta x + 1029x^{18}\Delta x^2 + 1029x^{17}\Delta x^3 + 500x^{16}\Delta x^4 + 21x^{15}\Delta x^5 + \Delta x^{20}) = 21x^{20}$
 $(x^{21})' = 21x^{20}$

$\Delta y = (x + \Delta x)^{22} - x^{22} = 22x^{21}\Delta x + 231x^{20}\Delta x^2 + 1122x^{19}\Delta x^3 + 1122x^{18}\Delta x^4 + 561x^{17}\Delta x^5 + 22x^{16}\Delta x^6 + \Delta x^{22}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (22x^{21} + 231x^{20}\Delta x + 1122x^{19}\Delta x^2 + 1122x^{18}\Delta x^3 + 561x^{17}\Delta x^4 + 22x^{16}\Delta x^5 + \Delta x^{21}) = 22x^{21}$
 $(x^{22})' = 22x^{21}$

$\Delta y = (x + \Delta x)^{23} - x^{23} = 23x^{22}\Delta x + 253x^{21}\Delta x^2 + 1254x^{20}\Delta x^3 + 1254x^{19}\Delta x^4 + 615x^{18}\Delta x^5 + 23x^{17}\Delta x^6 + \Delta x^{23}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (23x^{22} + 253x^{21}\Delta x + 1254x^{20}\Delta x^2 + 1254x^{19}\Delta x^3 + 615x^{18}\Delta x^4 + 23x^{17}\Delta x^5 + \Delta x^{22}) = 23x^{22}$
 $(x^{23})' = 23x^{22}$

$\Delta y = (x + \Delta x)^{24} - x^{24} = 24x^{23}\Delta x + 276x^{22}\Delta x^2 + 1380x^{21}\Delta x^3 + 1380x^{20}\Delta x^4 + 680x^{19}\Delta x^5 + 24x^{18}\Delta x^6 + \Delta x^{24}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (24x^{23} + 276x^{22}\Delta x + 1380x^{21}\Delta x^2 + 1380x^{20}\Delta x^3 + 680x^{19}\Delta x^4 + 24x^{18}\Delta x^5 + \Delta x^{23}) = 24x^{23}$
 $(x^{24})' = 24x^{23}$

$\Delta y = (x + \Delta x)^{25} - x^{25} = 25x^{24}\Delta x + 300x^{23}\Delta x^2 + 1500x^{22}\Delta x^3 + 1500x^{21}\Delta x^4 + 750x^{20}\Delta x^5 + 25x^{19}\Delta x^6 + \Delta x^{25}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (25x^{24} + 300x^{23}\Delta x + 1500x^{22}\Delta x^2 + 1500x^{21}\Delta x^3 + 750x^{20}\Delta x^4 + 25x^{19}\Delta x^5 + \Delta x^{24}) = 25x^{24}$
 $(x^{25})' = 25x^{24}$

$\Delta y = (x + \Delta x)^{26} - x^{26} = 26x^{25}\Delta x + 325x^{24}\Delta x^2 + 1625x^{23}\Delta x^3 + 1625x^{22}\Delta x^4 + 800x^{21}\Delta x^5 + 26x^{20}\Delta x^6 + \Delta x^{26}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (26x^{25} + 325x^{24}\Delta x + 1625x^{23}\Delta x^2 + 1625x^{22}\Delta x^3 + 800x^{21}\Delta x^4 + 26x^{20}\Delta x^5 + \Delta x^{25}) = 26x^{25}$
 $(x^{26})' = 26x^{25}$

$\Delta y = (x + \Delta x)^{27} - x^{27} = 27x^{26}\Delta x + 351x^{25}\Delta x^2 + 1755x^{24}\Delta x^3 + 1755x^{23}\Delta x^4 + 855x^{22}\Delta x^5 + 27x^{21}\Delta x^6 + \Delta x^{27}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (27x^{26} + 351x^{25}\Delta x + 1755x^{24}\Delta x^2 + 1755x^{23}\Delta x^3 + 855x^{22}\Delta x^4 + 27x^{21}\Delta x^5 + \Delta x^{26}) = 27x^{26}$
 $(x^{27})' = 27x^{26}$

$\Delta y = (x + \Delta x)^{28} - x^{28} = 28x^{27}\Delta x + 378x^{26}\Delta x^2 + 1890x^{25}\Delta x^3 + 1890x^{24}\Delta x^4 + 910x^{23}\Delta x^5 + 28x^{22}\Delta x^6 + \Delta x^{28}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (28x^{27} + 378x^{26}\Delta x + 1890x^{25}\Delta x^2 + 1890x^{24}\Delta x^3 + 910x^{23}\Delta x^4 + 28x^{22}\Delta x^5 + \Delta x^{27}) = 28x^{27}$
 $(x^{28})' = 28x^{27}$

$\Delta y = (x + \Delta x)^{29} - x^{29} = 29x^{28}\Delta x + 406x^{27}\Delta x^2 + 2030x^{26}\Delta x^3 + 2030x^{25}\Delta x^4 + 980x^{24}\Delta x^5 + 29x^{23}\Delta x^6 + \Delta x^{29}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (29x^{28} + 406x^{27}\Delta x + 2030x^{26}\Delta x^2 + 2030x^{25}\Delta x^3 + 980x^{24}\Delta x^4 + 29x^{23}\Delta x^5 + \Delta x^{28}) = 29x^{28}$
 $(x^{29})' = 29x^{28}$

$\Delta y = (x + \Delta x)^{30} - x^{30} = 30x^{29}\Delta x + 435x^{28}\Delta x^2 + 2142x^{27}\Delta x^3 + 2142x^{26}\Delta x^4 + 1020x^{25}\Delta x^5 + 30x^{24}\Delta x^6 + \Delta x^{30}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (30x^{29} + 435x^{28}\Delta x + 2142x^{27}\Delta x^2 + 2142x^{26}\Delta x^3 + 1020x^{25}\Delta x^4 + 30x^{24}\Delta x^5 + \Delta x^{29}) = 30x^{29}$
 $(x^{30})' = 30x^{29}$

$\Delta y = (x + \Delta x)^{31} - x^{31} = 31x^{30}\Delta x + 465x^{29}\Delta x^2 + 2277x^{28}\Delta x^3 + 2277x^{27}\Delta x^4 + 1080x^{26}\Delta x^5 + 31x^{25}\Delta x^6 + \Delta x^{31}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (31x^{30} + 465x^{29}\Delta x + 2277x^{28}\Delta x^2 + 2277x^{27}\Delta x^3 + 1080x^{26}\Delta x^4 + 31x^{25}\Delta x^5 + \Delta x^{30}) = 31x^{30}$
 $(x^{31})' = 31x^{30}$

$\Delta y = (x + \Delta x)^{32} - x^{32} = 32x^{31}\Delta x + 496x^{30}\Delta x^2 + 2432x^{29}\Delta x^3 + 2432x^{28}\Delta x^4 + 1120x^{27}\Delta x^5 + 32x^{26}\Delta x^6 + \Delta x^{32}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (32x^{31} + 496x^{30}\Delta x + 2432x^{29}\Delta x^2 + 2432x^{28}\Delta x^3 + 1120x^{27}\Delta x^4 + 32x^{26}\Delta x^5 + \Delta x^{31}) = 32x^{31}$
 $(x^{32})' = 32x^{31}$

$\Delta y = (x + \Delta x)^{33} - x^{33} = 33x^{32}\Delta x + 528x^{31}\Delta x^2 + 2601x^{30}\Delta x^3 + 2601x^{29}\Delta x^4 + 1155x^{28}\Delta x^5 + 33x^{27}\Delta x^6 + \Delta x^{33}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (33x^{32} + 528x^{31}\Delta x + 2601x^{30}\Delta x^2 + 2601x^{29}\Delta x^3 + 1155x^{28}\Delta x^4 + 33x^{27}\Delta x^5 + \Delta x^{32}) = 33x^{32}$
 $(x^{33})' = 33x^{32}$

$\Delta y = (x + \Delta x)^{34} - x^{34} = 34x^{33}\Delta x + 562x^{32}\Delta x^2 + 2786x^{31}\Delta x^3 + 2786x^{30}\Delta x^4 + 1188x^{29}\Delta x^5 + 34x^{28}\Delta x^6 + \Delta x^{34}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (34x^{33} + 562x^{32}\Delta x + 2786x^{31}\Delta x^2 + 2786x^{30}\Delta x^3 + 1188x^{29}\Delta x^4 + 34x^{28}\Delta x^5 + \Delta x^{33}) = 34x^{33}$
 $(x^{34})' = 34x^{33}$

$\Delta y = (x + \Delta x)^{35} - x^{35} = 35x^{34}\Delta x + 595x^{33}\Delta x^2 + 2989x^{32}\Delta x^3 + 2989x^{31}\Delta x^4 + 1210x^{30}\Delta x^5 + 35x^{29}\Delta x^6 + \Delta x^{35}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (35x^{34} + 595x^{33}\Delta x + 2989x^{32}\Delta x^2 + 2989x^{31}\Delta x^3 + 1210x^{30}\Delta x^4 + 35x^{29}\Delta x^5 + \Delta x^{34}) = 35x^{34}$
 $(x^{35})' = 35x^{34}$

$\Delta y = (x + \Delta x)^{36} - x^{36} = 36x^{35}\Delta x + 630x^{34}\Delta x^2 + 3204x^{33}\Delta x^3 + 3204x^{32}\Delta x^4 + 1224x^{31}\Delta x^5 + 36x^{30}\Delta x^6 + \Delta x^{36}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (36x^{35} + 630x^{34}\Delta x + 3204x^{33}\Delta x^2 + 3204x^{32}\Delta x^3 + 1224x^{31}\Delta x^4 + 36x^{30}\Delta x^5 + \Delta x^{35}) = 36x^{35}$
 $(x^{36})' = 36x^{35}$

$\Delta y = (x + \Delta x)^{37} - x^{37} = 37x^{36}\Delta x + 666x^{35}\Delta x^2 + 3432x^{34}\Delta x^3 + 3432x^{33}\Delta x^4 + 1230x^{32}\Delta x^5 + 37x^{31}\Delta x^6 + \Delta x^{37}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (37x^{36} + 666x^{35}\Delta x + 3432x^{34}\Delta x^2 + 3432x^{33}\Delta x^3 + 1230x^{32}\Delta x^4 + 37x^{31}\Delta x^5 + \Delta x^{36}) = 37x^{36}$
 $(x^{37})' = 37x^{36}$

$\Delta y = (x + \Delta x)^{38} - x^{38} = 38x^{37}\Delta x + 702x^{36}\Delta x^2 + 3672x^{35}\Delta x^3 + 3672x^{34}\Delta x^4 + 1230x^{33}\Delta x^5 + 38x^{32}\Delta x^6 + \Delta x^{38}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (38x^{37} + 702x^{36}\Delta x + 3672x^{35}\Delta x^2 + 3672x^{34}\Delta x^3 + 1230x^{33}\Delta x^4 + 38x^{32}\Delta x^5 + \Delta x^{37}) = 38x^{37}$
 $(x^{38})' = 38x^{37}$

$\Delta y = (x + \Delta x)^{39} - x^{39} = 39x^{38}\Delta x + 739x^{37}\Delta x^2 + 3927x^{36}\Delta x^3 + 3927x^{35}\Delta x^4 + 1230x^{34}\Delta x^5 + 39x^{33}\Delta x^6 + \Delta x^{39}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (39x^{38} + 739x^{37}\Delta x + 3927x^{36}\Delta x^2 + 3927x^{35}\Delta x^3 + 1230x^{34}\Delta x^4 + 39x^{33}\Delta x^5 + \Delta x^{38}) = 39x^{38}$
 $(x^{39})' = 39x^{38}$

$\Delta y = (x + \Delta x)^{40} - x^{40} = 40x^{39}\Delta x + 776x^{38}\Delta x^2 + 4180x^{37}\Delta x^3 + 4180x^{36}\Delta x^4 + 1230x^{35}\Delta x^5 + 40x^{34}\Delta x^6 + \Delta x^{40}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (40x^{39} + 776x^{38}\Delta x + 4180x^{37}\Delta x^2 + 4180x^{36}\Delta x^3 + 1230x^{35}\Delta x^4 + 40x^{34}\Delta x^5 + \Delta x^{39}) = 40x^{39}$
 $(x^{40})' = 40x^{39}$

$\Delta y = (x + \Delta x)^{41} - x^{41} = 41x^{40}\Delta x + 814x^{39}\Delta x^2 + 4441x^{38}\Delta x^3 + 4441x^{37}\Delta x^4 + 1230x^{36}\Delta x^5 + 41x^{35}\Delta x^6 + \Delta x^{41}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (41x^{40} + 814x^{39}\Delta x + 4441x^{38}\Delta x^2 + 4441x^{37}\Delta x^3 + 1230x^{36}\Delta x^4 + 41x^{35}\Delta x^5 + \Delta x^{40}) = 41x^{40}$
 $(x^{41})' = 41x^{40}$

$\Delta y = (x + \Delta x)^{42} - x^{42} = 42x^{41}\Delta x + 854x^{40}\Delta x^2 + 4710x^{39}\Delta x^3 + 4710x^{38}\Delta x^4 + 1230x^{37}\Delta x^5 + 42x^{36}\Delta x^6 + \Delta x^{42}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (42x^{41} + 854x^{40}\Delta x + 4710x^{39}\Delta x^2 + 4710x^{38}\Delta x^3 + 1230x^{37}\Delta x^4 + 42x^{36}\Delta x^5 + \Delta x^{41}) = 42x^{41}$
 $(x^{42})' = 42x^{40}$

$\Delta y = (x + \Delta x)^{43} - x^{43} = 43x^{42}\Delta x + 895x^{41}\Delta x^2 + 4989x^{40}\Delta x^3 + 4989x^{39}\Delta x^4 + 1230x^{38}\Delta x^5 + 43x^{37}\Delta x^6 + \Delta x^{43}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (43x^{42} + 895x^{41}\Delta x + 4989x^{40}\Delta x^2 + 4989x^{39}\Delta x^3 + 1230x^{38}\Delta x^4 + 43x^{37}\Delta x^5 + \Delta x^{42}) = 43x^{42}$
 $(x^{43})' = 43x^{41}$

$\Delta y = (x + \Delta x)^{44} - x^{44} = 44x^{43}\Delta x + 936x^{42}\Delta x^2 + 5276x^{41}\Delta x^3 + 5276x^{40}\Delta x^4 + 1230x^{39}\Delta x^5 + 44x^{38}\Delta x^6 + \Delta x^{44}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (44x^{43} + 936x^{42}\Delta x + 5276x^{41}\Delta x^2 + 5276x^{40}\Delta x^3 + 1230x^{39}\Delta x^4 + 44x^{38}\Delta x^5 + \Delta x^{43}) = 44x^{43}$
 $(x^{44})' = 44x^{42}$

$\Delta y = (x + \Delta x)^{45} - x^{45} = 45x^{44}\Delta x + 978x^{43}\Delta x^2 + 5571x^{42}\Delta x^3 + 5571x^{41}\Delta x^4 + 1230x^{40}\Delta x^5 + 45x^{39}\Delta x^6 + \Delta x^{45}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (45x^{44} + 978x^{43}\Delta x + 5571x^{42}\Delta x^2 + 5571x^{41}\Delta x^3 + 1230x^{40}\Delta x^4 + 45x^{39}\Delta x^5 + \Delta x^{44}) = 45x^{44}$
 $(x^{45})' = 45x^{43}$

$\Delta y = (x + \Delta x)^{46} - x^{46} = 46x^{45}\Delta x + 1020x^{44}\Delta x^2 + 5874x^{43}\Delta x^3 + 5874x^{42}\Delta x^4 + 1230x^{41}\Delta x^5 + 46x^{40}\Delta x^6 + \Delta x^{46}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (46x^{45} + 1020x^{44}\Delta x + 5874x^{43}\Delta x^2 + 5874x^{42}\Delta x^3 + 1230x^{41}\Delta x^4 + 46x^{40}\Delta x^5 + \Delta x^{45}) = 46x^{45}$
 $(x^{46})' = 46x^{44}$

$\Delta y = (x + \Delta x)^{47} - x^{47} = 47x^{46}\Delta x + 1064x^{45}\Delta x^2 + 6185x^{44}\Delta x^3 + 6185x^{43}\Delta x^4 + 1230x^{42}\Delta x^5 + 47x^{41}\Delta x^6 + \Delta x^{47}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (47x^{46} + 1064x^{45}\Delta x + 6185x^{44}\Delta x^2 + 6185x^{43}\Delta x^3 + 1230x^{42}\Delta x^4 + 47x^{41}\Delta x^5 + \Delta x^{46}) = 47x^{46}$
 $(x^{47})' = 47x^{45}$

$\Delta y = (x + \Delta x)^{48} - x^{48} = 48x^{47}\Delta x + 1108x^{46}\Delta x^2 + 6504x^{45}\Delta x^3 + 6504x^{44}\Delta x^4 + 1230x^{43}\Delta x^5 + 48x^{42}\Delta x^6 + \Delta x^{48}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (48x^{47} + 1108x^{46}\Delta x + 6504x^{45}\Delta x^2 + 6504x^{44}\Delta x^3 + 1230x^{43}\Delta x^4 + 48x^{42}\Delta x^5 + \Delta x^{47}) = 48x^{47}$
 $(x^{48})' = 48x^{46}$

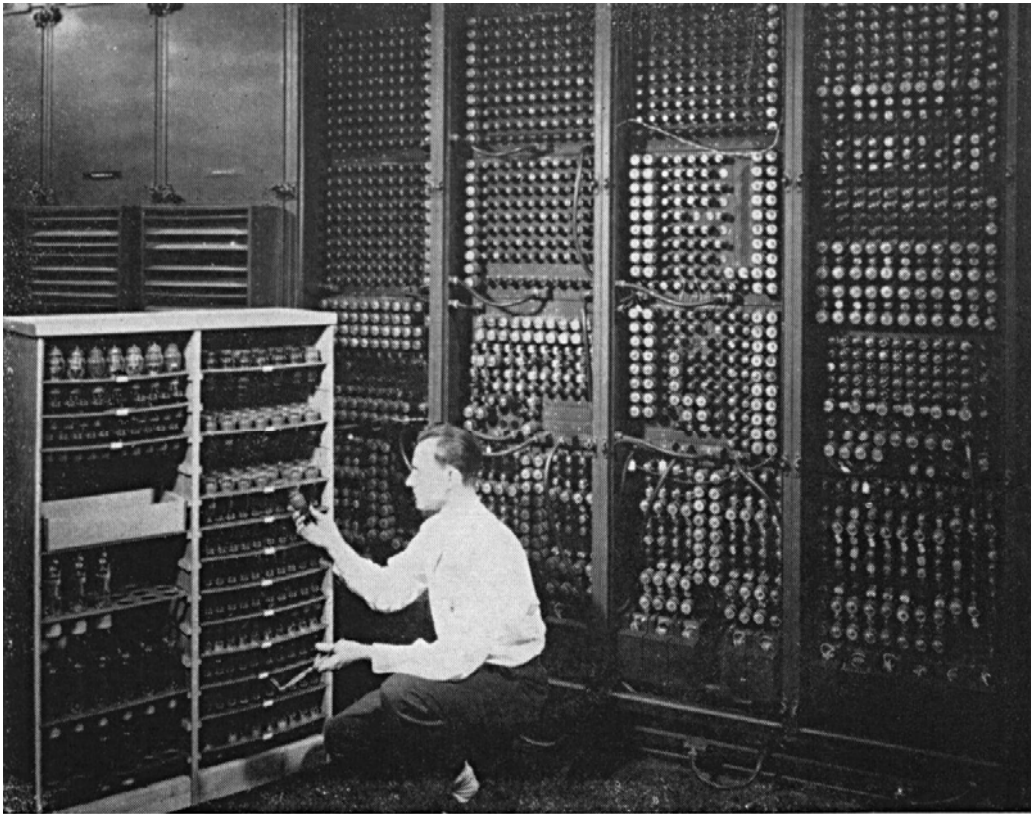
$\Delta y = (x + \Delta x)^{49} - x^{49} = 49x^{48}\Delta x + 1154x^{47}\Delta x^2 + 6831x^{46}\Delta x^3 + 6831x^{45}\Delta x^4 + 1230x^{44}\Delta x^5 + 49x^{43}\Delta x^6 + \Delta x^{49}$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (49x^{48} + 1154x^{47}\Delta x + 6831x^{46}\Delta x^2 + 6831x^{45}\Delta x^3 + 1230x^{44}\Delta x^4 + 49x^{43}\Delta x^5 + \Delta x^{48}) = 49x^{48}$
 $(x^{49})' = 49x^{47}$

$\Delta y = (x + \Delta x)^{50} - x^{50} = 50x^{49}\Delta x + 1200x^{48}\Delta x^2 + 7166x^{47}\Delta x^3 + 7166x^{46}\Delta x^4 + 1230x^{45}\Delta x^5 + 50x^{44}\Delta x^6 + \Delta x^{50}$
 $\lim_{\Delta x \rightarrow$

Et jusqu'à tout récemment, on a procédé de cette façon.

- ▶ À un phénomène donné, on peut calculer la trajectoire d'une planète, l'heure des marées (et la trajectoire d'un obus)...
- ▶ Et arriva l'ENIAC en 1948 : Sa première tâche a été de :
- ▶ Calculer le nombre $\pi = 3,14159 \dots$ et le nombre $e = 2,71828182845 \dots$ (base des logarithmes naturels) à 2000 décimales.
- ▶ ENIAC = Electronic Numerical Integrator Analyzer and Calculator.
- ▶ *17468 tubes, 70000 résistances, 10000 condensateurs.*
- ▶ *30 tonnes, $2,4 \times 0,9 \times 30,5$ mètres, sur 167 mètres carrés et qui bouffe 150 KW d'électricité, un monstre.*
- ▶ *5000 additions par seconde, 357 multiplications ou 38 divisions par seconde (seulement).*

ENIAC 1948



Replacing a bad tube meant checking among ENIAC's 19,000 possibilities.

Malgré sa taille, il calculait vite...

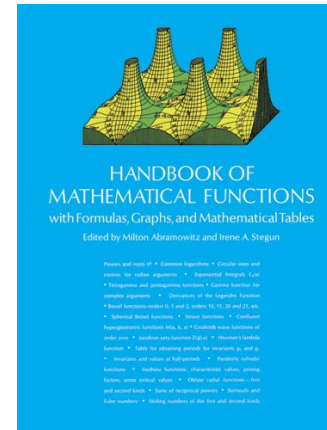
Moyen employé	Vitesse de multiplication de nombres de 10 chiffres	Calcul d'une trajectoire d'une table de tir
Homme à la main, ou <i>machine de Babbage</i>	5 min	2,6 jours
Homme avec calculateur de bureau	10 à 15 secondes	12 heures
<i>Harvard Mark I</i> (électromécanique)	3 secondes	2 heures
Model 5 (électromécanique)	2 secondes	40 minutes
Analyseur différentiel (analogique)	1 seconde	20 minutes
<i>Harvard Mark II</i> (électromécanique)	0,4 s	15 minutes
ENIAC (électronique)	0,001 s	3 secondes

Table extraite du livre 'Handbook of Mathematical Functions 1964

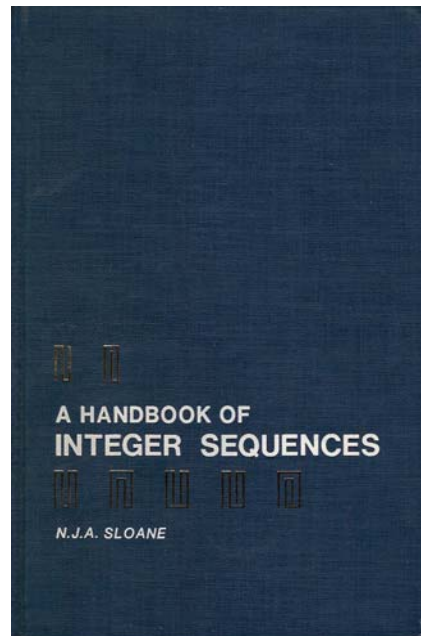
MATHEMATICAL CONSTANTS

TABLE I.1. MATHEMATICAL CONSTANTS—Continued

n	$\ln n$				n	$\log_{10} n$					
47	3.8501	47601	71005	85868	209507	47	1.6720	97857	93571	74644	14219
53	3.9702	91913	55212	18341	444691	53	1.7242	75869	60078	90456	32992
59	4.0775	37443	90571	94506	160504	59	1.7708	52011	64214	41902	60656
61	4.1108	73864	17331	12487	513891	61	1.7853	29835	01076	70338	85749
67	4.2046	92619	39096	60596	700720	67	1.8260	74802	70082	64341	49132
71	4.2626	79877	04131	54213	294545	71	1.8512	58348	71907	52860	92829
73	4.2904	59441	14839	11290	921089	73	1.8633	22860	12045	59010	74387
79	4.3694	47852	46702	14941	729455	79	1.8976	27091	29044	14279	94821
83	4.4188	40607	79659	79234	754722	83	1.9190	78092	37607	39038	32760
89	4.4886	36369	73213	98383	178155	89	1.9493	90006	64491	27847	23543
97	4.5747	10978	50338	28221	167216	97	1.9867	71734	70624	48517	84362
$\ln \pi$	1.1447	29885	84940	01741	43427	$\log_{10} \pi$	(-1) 4.9714	98726	94133	85435	12683
$\ln \sqrt{2} \pi$	(-1) 9.1893	85332	04672	74178	03296	$\log_{10} e$	(-1) 4.3429	44819	03251	82765	11289
n	$n \ln 10$				n	$n \pi$					
1	2.3025	85092	99404	56840	17991	1	3.1415	92653	58979	32384	62643
2	4.6051	70185	98809	13680	35983	2	6.2831	85307	17958	64769	25287
3	6.9077	55278	98213	70520	53974	3	9.4247	77960	76937	97153	87930
4	9.2103	40371	97618	27360	71966	4	(1) 1.2566	37061	43591	72953	85057
5	(1) 1.1512	92546	49702	28420	08996	5	(1) 1.5707	96326	79489	66192	31322
6	(1) 1.3815	51055	79642	74104	10795	6	(1) 1.8849	55592	15387	59430	77586
7	(1) 1.6118	09565	09583	19788	12594	7	(1) 2.1991	14857	51285	52669	23850
8	(1) 1.8420	68074	39523	65472	14393	8	(1) 2.5132	74122	87183	45907	70115
9	(1) 2.0723	26583	69464	11156	16192	9	(1) 2.8274	33388	23081	39146	16379
n	π^n				n	π^{-n}					
1	3.1415	92653	58979	32384	62643	1	(-1) 3.1830	98861	83790	67153	77675
2	9.8696	04401	08935	86188	34491	2	(-1) 1.0132	11836	42337	77144	38795
3	(1) 3.1006	27668	02998	20175	47632	3	(-2) 3.2251	53443	31994	89184	42205
4	(1) 9.7409	09103	40024	37236	44033	4	(-2) 1.0265	98225	46843	35189	15278
5	(2) 3.0601	96847	85281	45326	27413	5	(-3) 3.2677	63643	05338	54726	28250
6	(2) 9.6138	91935	75304	43703	02194	6	(-3) 1.0401	61473	29585	22960	89838
7	(3) 3.0202	93227	77679	20675	14206	7	(-4) 3.3109	36801	77566	76432	59528
8	(3) 9.4885	31016	07057	40071	28576	8	(-4) 1.0539	03916	53493	66633	17287
9	(4) 2.9809	09933	34462	11666	50940	9	(-5) 3.3546	80357	20886	91287	39854
10	(4) 9.3648	04747	60830	20973	71669	10	(-5) 1.0678	27922	68615	33662	04078



En 1973, Neil J.A. Sloane publie 200 pages de chiffres



361 1, 2, 4, 5, 8, 9, 10, 13, 16, 17, 18, 20, 25, 26, 29, 32, 34, 36, 37, 40, 41, 45, 49, 50, 52, 53, 58, 61, 64, 65, 68, 72, 73, 74, 80, 81, 82, 85, 89, 90, 97, 98, 100, 101, 104, 106
THE SUM OF 2 SQUARES. REF EUL (1) 1 417 11. KNAW 53 872 50.

362 1, 2, 4, 5, 8, 9, 12, 14, 17, 18, 23, 24, 27, 30, 34, 35, 40, 41, 46, 49, 52, 53, 60, 62, 65, 68, 73, 74, 81, 82, 87, 90, 93, 96, 104, 105, 108, 111, 118, 119, 126, 127, 132, 137
A NUMBER-THEORETIC FUNCTION. REF DVSS 2 281 1884.

363 1, 2, 4, 5, 8, 10, 14, 15, 16, 21, 22, 25, 26, 28, 33, 34, 35, 36, 38, 40, 42, 46, 48, 49, 50, 53, 57, 60, 62, 64, 65, 70, 77, 80, 81, 83, 85, 86, 90, 91, 92, 100
PRIME NUMBERS OF MEASUREMENT. REF PCPS 21 654 23.

364 1, 2, 4, 5, 8, 12, 19, 30, 48, 77, 124, 200, 323, 522, 844, 1365, 2208, 3572, 5779, 9350, 15128, 24477, 39604, 64080, 103683, 167762, 271444, 439205, 710648, 1149852
 $A(N) = A(N - 1) + A(N - 2) - 1$. REF JA2 97.

365 1, 2, 4, 5, 10, 14, 17, 31, 41, 73, 80, 82, 116, 125, 145, 157, 172, 202, 224, 266, 289, 293, 463
 $15.2 \cdot N - 1$ IS PRIME. REF MTAC 23 874 69.

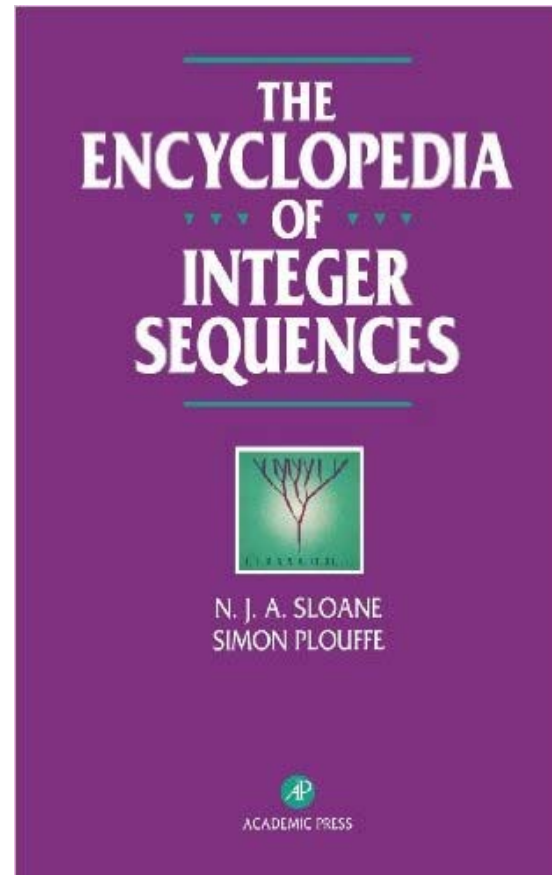
366 1, 2, 4, 5, 10, 19, 36, 68, 138
BORON TREES. REF CAY 9 451.

367 1, 2, 4, 5, 14, 14, 39, 42, 132, 132, 424, 429, 1428, 1430, 4848, 4862, 16796, 16796, 58739, 58786, 208012, 208012, 742768, 742900, 2674426, 2674440, 9694416
DISSECTIONS OF A POLYGON. REF GU1.

368 1, 2, 4, 6, 3, 10, 25, 12, 42, 8, 40, 202, 21
FROM SEDLACEKS PROBLEM ON SOLUTIONS OF $X + Y = Z$. REF GU8.

369 1, 2, 4, 6, 7, 10, 11, 12, 22, 23, 25, 26, 27, 30, 36, 38, 42, 43, 44, 45, 50, 52, 54, 58, 59, 70, 71, 72, 74, 75, 76, 78, 86, 87, 91, 102, 103, 106, 107, 108, 110, 116, 118, 119
ELLIPTIC CURVES. REF JRAM 212 25 63.

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Mais en fait, toutes ces formules (et les tables qui viennent avec) sont toutes faites de la même façon.

- ▶ D'une formule donnée on a une série de chiffres qui représentent ce que la fonction est à ce point.

$$\frac{\pi e \lim_{n \rightarrow \infty} \frac{2^{2n} (n!)^2 \log 7}{(2n)! \sqrt{n}}}{\int_0^{\infty} e^{-t^2} dt} \frac{-e^t \sum_{k=0}^{\infty} \frac{8\pi}{(4k+1)(4k+3)}}{[\int_0^{\infty} \frac{3dt}{t^6+1}] [\int_{-\infty}^{\infty} e^{-\pi t^2} dt]} = 50$$

- ▶ Il y a la formule à gauche -> résultat ou série de chiffres à droite.





Mais si on pose la question : Comment faire pour aller de droite à gauche ?

Le Inverse Symbolic Calculator, Vancouver, juillet 1995 , 12 millions de constantes.

INVERSE SYMBOLIC CALCULATOR

Please enter a number or a Maple expression:

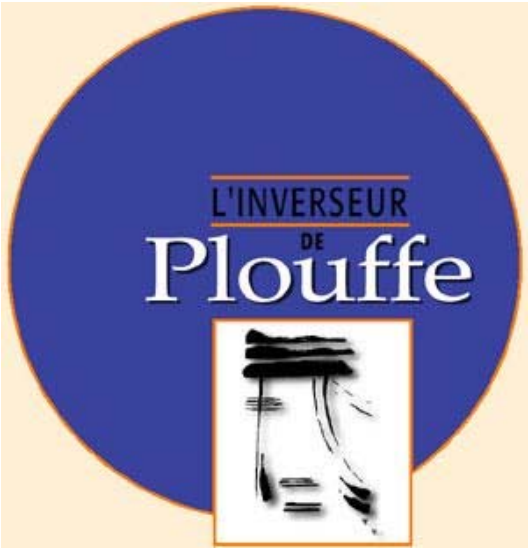
- **Simple Lookup and Browser** for any number.
- **Smart Lookup** for any number.
- **Generalized Expansions** for real numbers of at least 16 digits.
- **Integer Relation Algorithms** for any number.

Expressions that are **not** numeric like $\ln(\text{Pi}*\text{sqrt}(2))$ are evaluated in [Maple](#) in symbolic form first, followed by a floating point evaluation followed by a lookup.

3141591706703055 = (0131) $\sum(1/(2^n-2^{n+13}), n=1..inf)$
 3141591746153406 = (0258) $F(10/11, 5/11; 7/9, 4/9; 1)$
 3141591773236929 = (0261) $1+2*x-4*x^2+3*x^4+2*x^5$
 3141591936381883 = (0326) $11^{(1/3)}*(2^{(2/3)}-3)$
 3141592126650245 = (0131) $\sum(1/(37^n-30^{n+67}), n=1..inf)$
 3141592211352067 = (0001) $\text{Bernstein}^{\text{GAM}(1/12)}-\text{Pi}$
 3141592227548461 = (0001) $\text{Pi}-\exp(-\text{Pi})^{\text{Feig1}}$
 3141592434690729 = (0261) $-5+5*x+2*x^2+4*x^3-5*x^4-2*x^5$
 3141592475846874 = (0001) $\text{Pi}-\sin(\text{Pi}/12)^{\text{GAM}(1/12)}$
 3141592628153719 = (0001) $\text{Pi}-\exp(-\text{Pi})^{\text{GAM}(1/6)}$
 3141592639293414 = (0001) $\text{Pi}-\exp(-1/2*\text{Pi})^{\text{GAM}(1/12)}$
 3141592653589793 = (0000) Pi
 3141592653589793 = (0001) $1/2*\text{GAM}(1/6)*\text{GAM}(5/6)$
 3141592653589793 = (0001) $\text{GAM}(1/4)/\text{sr}(2)*\text{GAM}(3/4)$
 3141592653589793 = (0001) $\text{GAM}(1/6)*\text{GAM}(5/6)-\text{Pi}$
 3141592653589793 = (0001) $\cos(\text{Pi}/12)*\text{GAM}(5/12)*\text{GAM}(7/12)$
 3141592653589793 = (0001) $\exp(\text{gamma})^{(\ln(\text{Pi})/\text{gamma})}$
 3141592653589793 = (0001) $\sin(\text{Pi}/12)*\text{GAM}(11/12)*\text{GAM}(1/12)$
 3141592653589793 = (0141) $\text{Psi}(1/4)-\text{Psi}(3/4)$
 3141592653589793 = (0396) [A000796](#) from Enc. of Int. Seq.
 3141592653589793 = (0397) $\sum(\text{A007514}(n)/(n-1)!) \text{ from E.I.S.}$
 3141592653766556 = (0405) $\ln(\text{Parking}+\text{Madelung})$
 3141592667886172 = (0001) $\text{Pi}+\exp(-1/2*\text{Pi})^{\text{GAM}(1/12)}$
 3141592679025866 = (0001) $\text{Pi}+\exp(-\text{Pi})^{\text{GAM}(1/6)}$
 3141592790518436 = (0001) $\exp(\text{Pi})+\text{khint}*\text{GAM}(7/24)$
 3141592831332712 = (0001) $\text{Pi}+\sin(\text{Pi}/12)^{\text{GAM}(1/12)}$
 3141592920353982 = (0399) $71/226$
 3141593079631124 = (0001) $\exp(-\text{Pi})^{\text{Feig1}}+\text{Pi}$
 3141593095827519 = (0001) $\text{Pi}+\text{Bernstein}^{\text{GAM}(1/12)}$
 3141593167750063 = (0008) $\sum((8/3*n^3-31/2*n^2+287/6*n-16)/n^n, n=1..inf)$
 3141593615816239 = (0011) $\sum((5/2*n^3-4*n^2+7/2*n+21)/(n!+2), n=1..inf)$
 3141593656148726 = (0001) $\text{Pi}+\exp(-\text{Pi})^{\text{GAM}(5/24)}$
 3141593769945568 = (0001) $\text{Pi}+\text{GaussK}^{\text{GAM}(1/12)}$

1998, l'Inverseur de Plouffe
200 millions de constantes



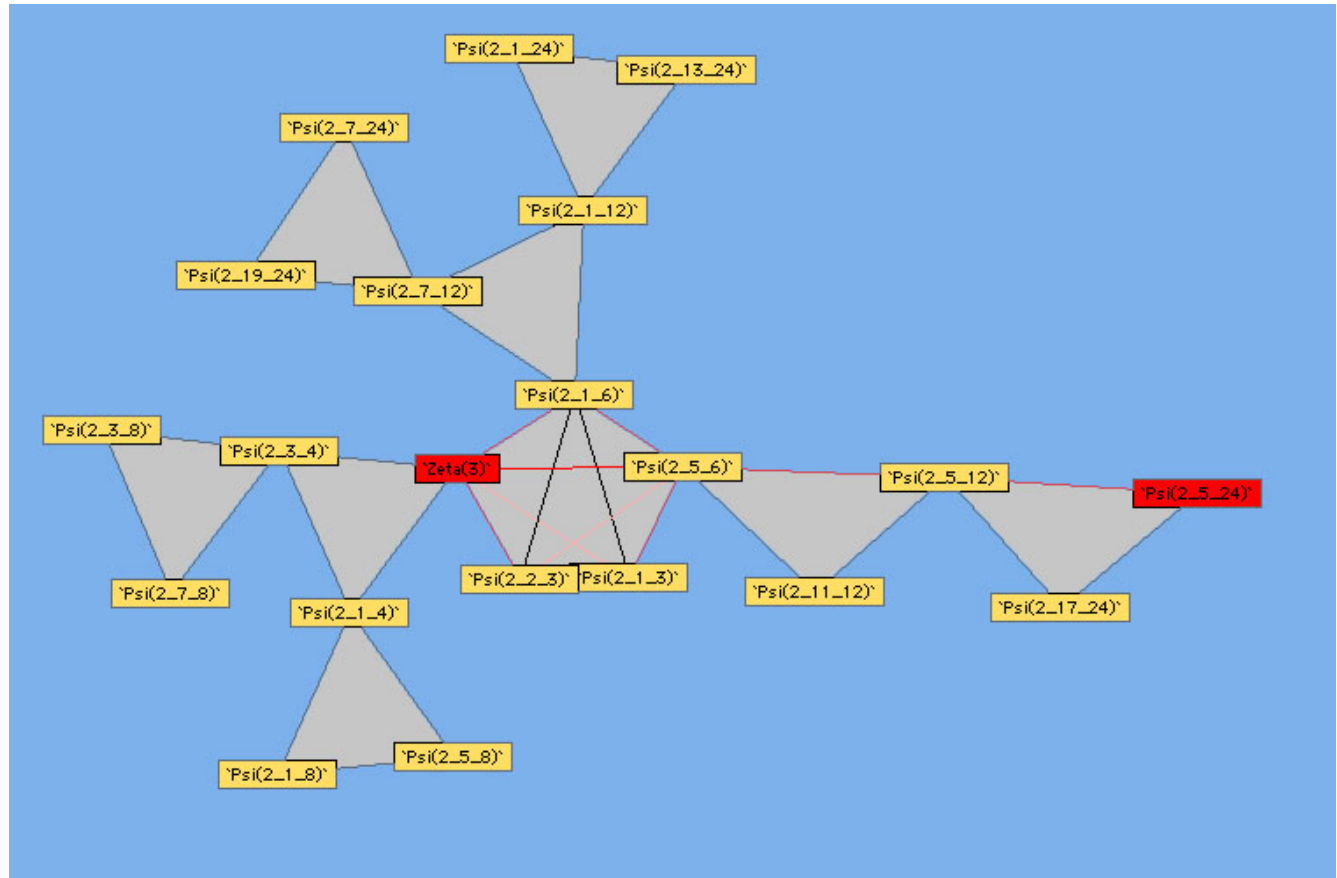
Copie en Australie 2001,
54 millions de constantes



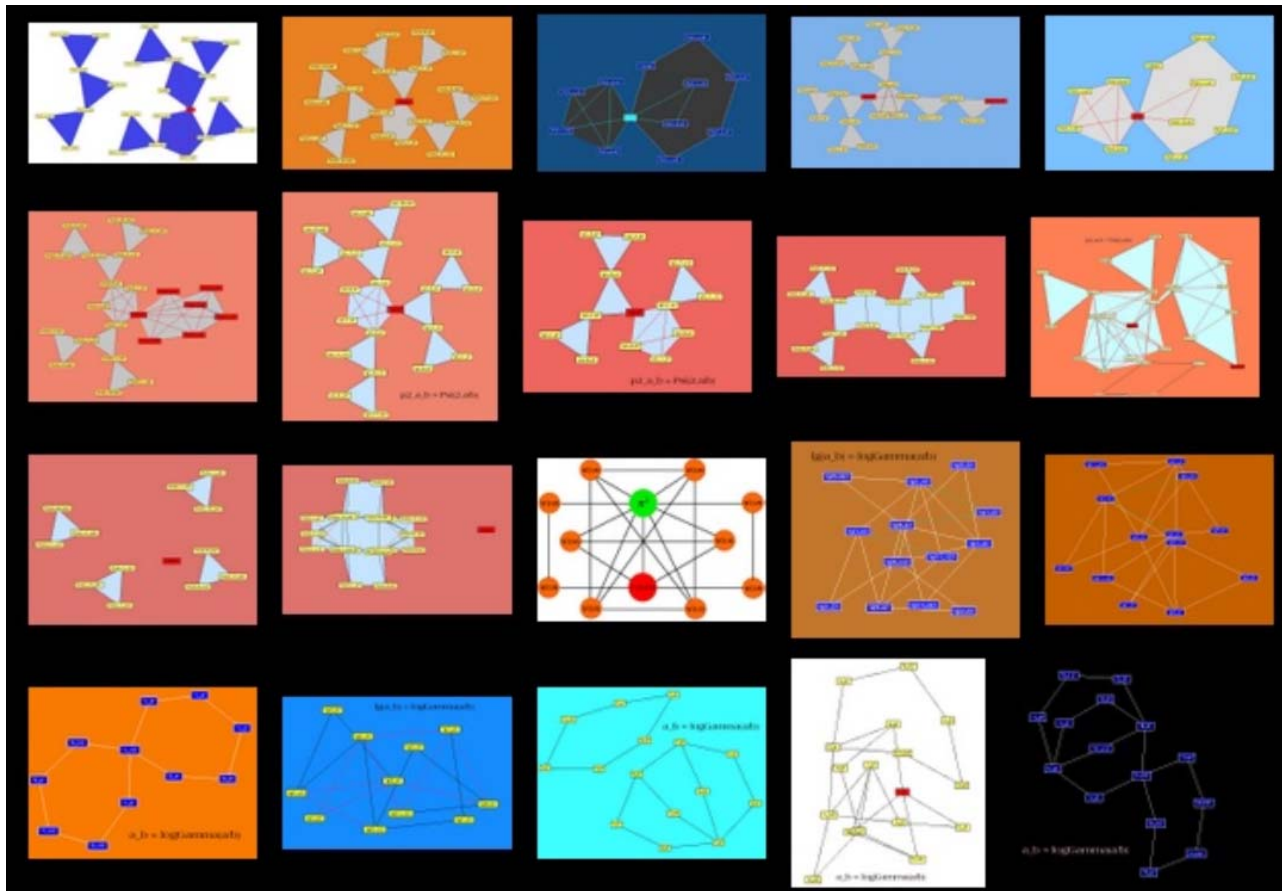
Inverseur de Plouffe (2016)

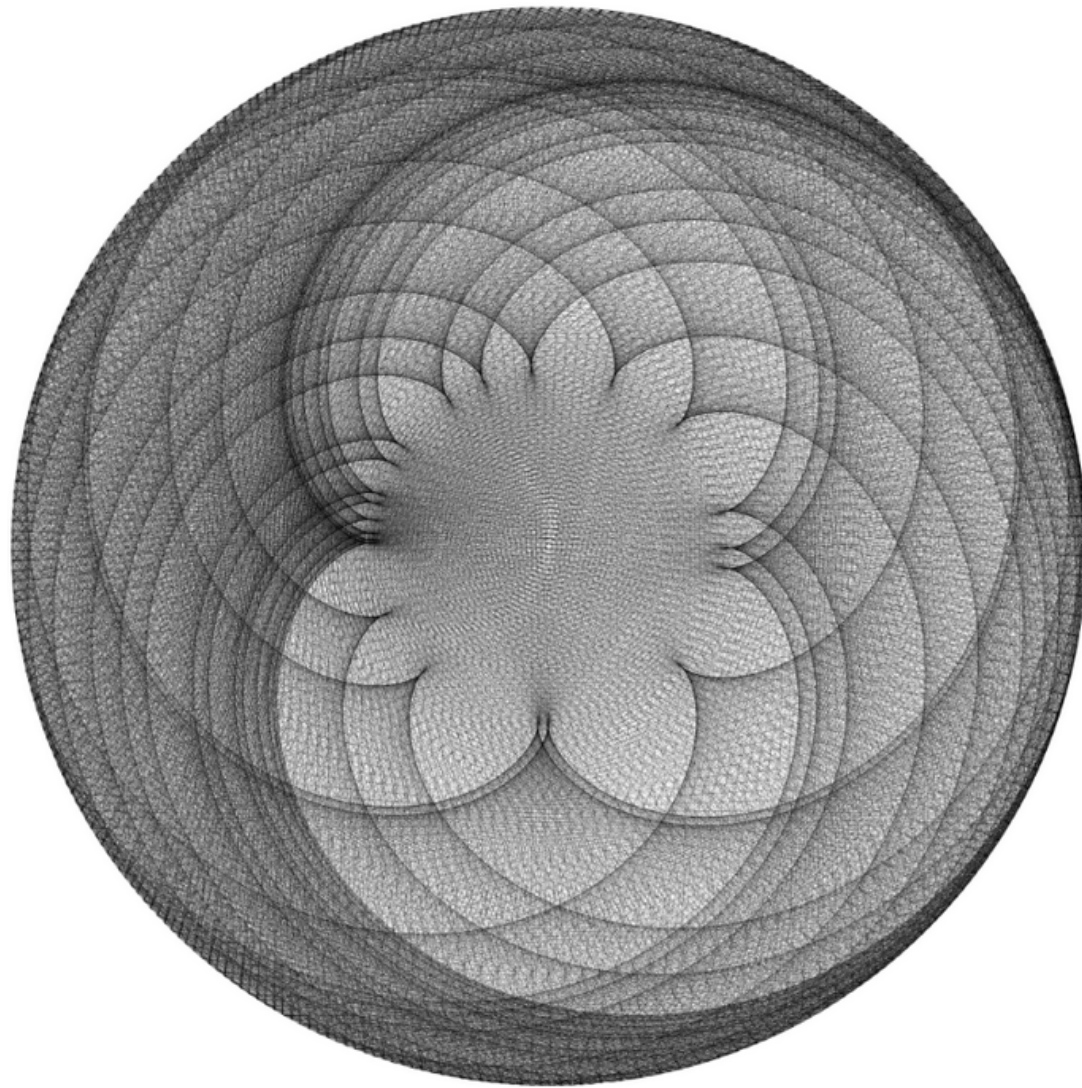
- ▶ 17,3 TB de chiffres (plusieurs inverseurs).
- ▶ 16 120 000 000 de constantes à 64 décimales
- ▶ 3,06 milliards de constantes à 32 décimales <http://plouffe.fr/ip/>
- ▶ Ça ressemble à ceci :

```
50320000257497674922482298719135 a003 sin(Pi*23/66)/cos(Pi*51/115)
50320000283141325986375419272560 s002 sum(A021891[n]/(binomial(2*n,n)*n^4*5^n),n=1..infinity)
50320000299457417829230177008734 a015 Real Root of -x^16-x^15+x^14-x^12+x^11-x^9-x^7-x^6-x^5-x^4-x^3-x^2+x
50320000441140589575284647039369 m001 (LambertW(1)-exp(1/Pi))/(-ErdosBorwein+ZetaQ(4))
50320000458534998108646748710839 v021 sum(1/binomial(2*n,n)/(7/2*n^3-35/2*n^2+88*n-73),n=1..infinity)
50320000507560798423029805602606 g005 GAMMA(1/11)*Pi*csc(1/10*Pi)/GAMMA(9/10)*GAMMA(3/7)/GAMMA(2/9)
50320000525349631557371992649512 r009 Im(z^3+c),c=-1/52+29/46*I,n=48
50320000566094356003815388714842 m020 7/3*exp(3*Pi)+91/2*exp(2*Pi)-791/6*exp(Pi)+92
50320000639625416554532600032438 h028 log(hypergeom([1/2,5/8],[7/6,31/12,31/12],1/8))
50320000649793744271048552683812 r009 Im(z^3+c),c=-1/4+20/33*I,n=22
50320000702546384845236101553430 m018 95/2*Pi^2+55/2*Pi-52
50320000760941316687852372680400 r009 Im(z^3+c),c=-13/42+31/50*I,n=23
50320000837943528041481632716199 a007 Real Root Of 376*x^4-648*x^3-838*x^2-943*x-369
50320000838554577529946451549654 m001 Backhouse*(BesselI(0,2)+Salem)
50320000875430626937067971794318 r005 Im(z^2+c),c=-71/54+1/56*I,n=11
50320000909938180692069058637998 r008 a(0)=5,k{-n^6,-87-31*n^3+38*n^2+48*n}
50320001019336409493709821248244 s002 sum(A221554[n]/(binomial(2*n,n)*n^4*5^n),n=1..infinity)
50320001125612129264306592050486 h028 log(hypergeom([1/10,1/8],[17/10,19/8,31/12],5/12))
50320001167159712473810165127033 m021 17/3/Pi^3+6/Pi^2-26/3/Pi+7
50320001273173208103138598165576 m001 Zeta(1,2)^2*Riemann2ndZero^2*exp(sin(Pi/12))
```

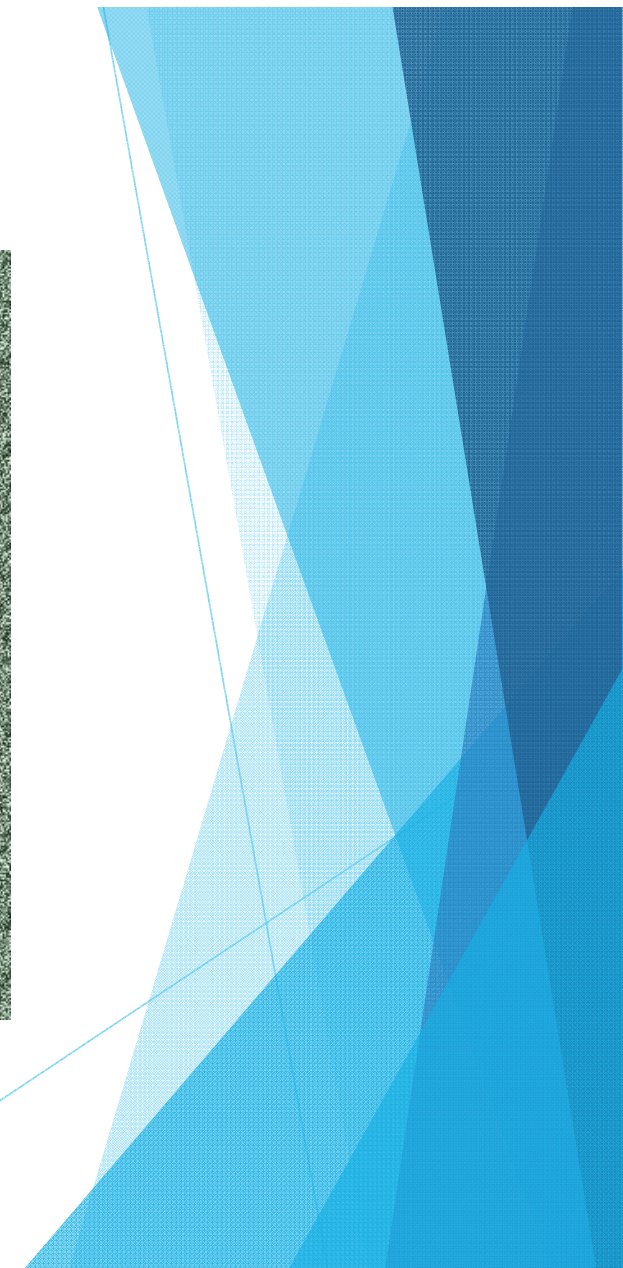


Et ça sert à trouver ça





Zéros de la fonction Zeta



Des formules (?) pour les masses des particules élémentaires.

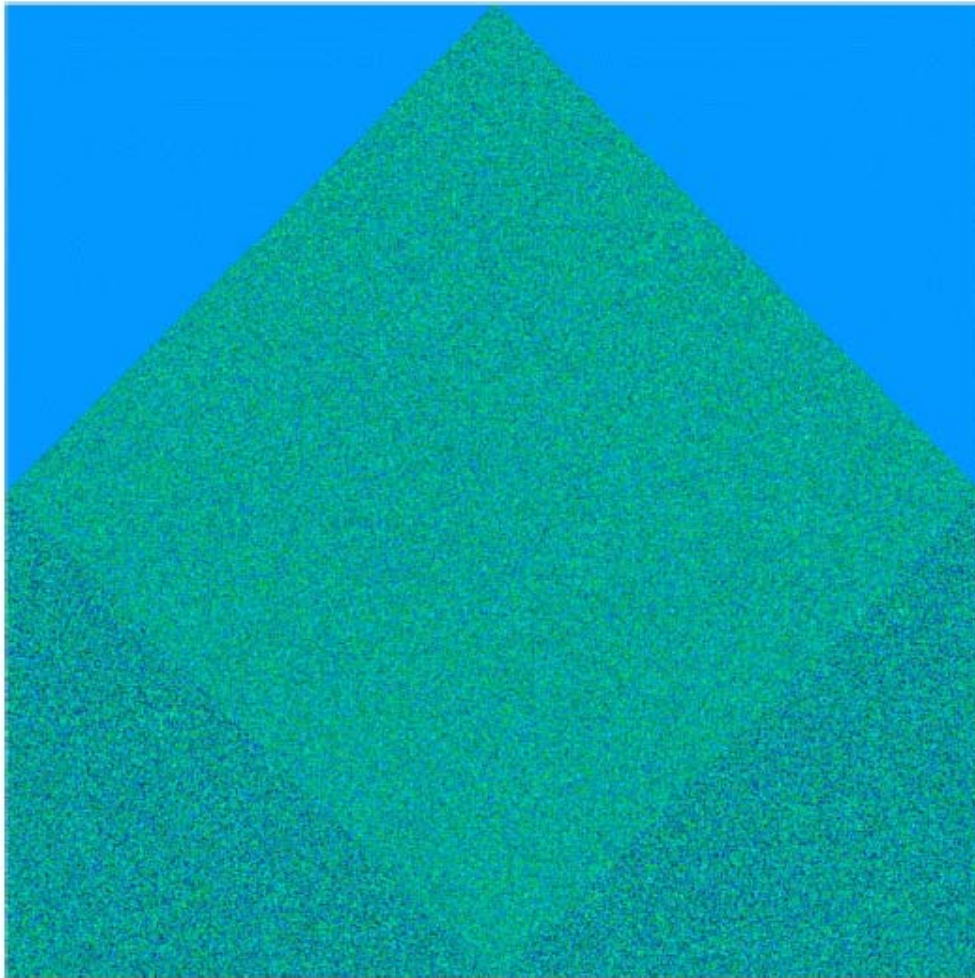
A formula for the proton/electron mass ratio:

$$\frac{1}{5 \cosh(\pi)} + 6 \pi^5 + \frac{1}{5 \sinh(\pi)} = 1836.15267996686153 \dots$$

$$\frac{M_n}{M_p} \approx \frac{8}{27} \left(\frac{5}{\cos\left(\frac{\pi}{15}\right)} - \sqrt{3} \right) = 1.001378419779635280 \dots$$

$$\cos\left(\frac{\pi}{15}\right) = \frac{1}{8} (-1 + \sqrt{5} + \sqrt{6(5 + \sqrt{5})})$$

Nombre algébrique à 270 millions de décimales (en binaire) particulier



$$f'(x) = 1 + \frac{\sqrt{2 \cdot 4^x + 2\sqrt{1.6^x + 1}}}{2 \cdot 2^x + 2}$$

Ou d'autres particulières

$$1992: e^\pi - \pi = 19.999099979 \dots$$

$$1995: \pi = \sum_{k=0}^{\infty} \frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right)$$

$$\sum_{n=1}^{\infty} \frac{n^3}{e^{2\pi n/7} - 1} = 10.0000000000000000190161767888663 \dots$$
$$\sum_{n=1}^{\infty} \frac{n^3}{e^{2\pi n/13} - 1} \cong 119.00000000000000000000000000000000000959374585 \dots$$

$$\pi = 72 \sum_{n=1}^{\infty} \frac{1}{n(e^{\pi n} - 1)} - 96 \sum_{n=1}^{\infty} \frac{1}{n(e^{2\pi n} - 1)} + 24 \sum_{n=1}^{\infty} \frac{1}{n(e^{4\pi n} - 1)}$$

$$\frac{1}{\pi} = 8 \sum_{n=1}^{\infty} \frac{n}{e^{\pi n} - 1} - 40 \sum_{n=1}^{\infty} \frac{n}{e^{2\pi n} - 1} + 32 \sum_{n=1}^{\infty} \frac{n}{e^{4\pi n} - 1}$$

$$\zeta(5) = \frac{694}{204813} \pi^5 - \frac{6280}{3251} \sum_{n=1}^{\infty} \frac{1}{n^5(e^{4\pi n} - 1)} + \frac{296}{3251} \sum_{n=1}^{\infty} \frac{1}{n^5(e^{5\pi n} - 1)} - \frac{1073}{6502} \sum_{n=1}^{\infty} \frac{1}{n^5(e^{10\pi n} - 1)} + \frac{37}{6502} \sum_{n=1}^{\infty} \frac{1}{n^5(e^{20\pi n} - 1)}$$

$$\zeta(5) = \frac{11\pi^5\sqrt{3}}{5670} + 2 \sum_{n=1}^{\infty} \frac{1}{n^5(e^{\sqrt{3}\pi n} - 1)} - \frac{33}{8} \sum_{n=1}^{\infty} \frac{1}{n^5(e^{\sqrt{12}\pi n} - 1)} + \frac{1}{8} \sum_{n=1}^{\infty} \frac{1}{n^5(e^{\sqrt{48}\pi n} - 1)}$$

$$\zeta(3) = \frac{13\pi^3\sqrt{3}}{45} + 2 \sum_{n=1}^{\infty} \frac{1}{n^3(e^{\sqrt{3}\pi n} - 1)} - \frac{9}{2} \sum_{n=1}^{\infty} \frac{1}{n^3(e^{2\sqrt{3}\pi n} - 1)} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n^3(e^{4\sqrt{3}\pi n} - 1)}$$

$$\zeta(5) = \frac{5\pi^5\sqrt{7}}{3906} + \frac{64}{31} \sum_{n=1}^{\infty} \frac{1}{n^5(e^{\sqrt{7}\pi n} - 1)} + \frac{130}{31} \sum_{n=1}^{\infty} \frac{1}{n^5(e^{\sqrt{28}\pi n} - 1)} - \frac{4}{31} \sum_{n=1}^{\infty} \frac{1}{n^5(e^{\sqrt{112}\pi n} - 1)}$$